Irradiation of Amyloid Fibrils with Free Electron Laser Tuned to Mid-Infrared Frequencies

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Abstract

In this work, the variation of plasma and cyclotronic frequencies on nonlinear permittivity ε_2 has been studied. Two equations of second order has been solved numerically using Runge-Kutta method. Decreasing the minimum beam width parameter is an important issue in some applications such as laser ignitor fusion, laser acceleration and harmonic generation. The effect of applying of magnetic field has been analyzed through the variation of minimum beam width parameter as a function of the cyclotronic frequency. The effect of magnetized plasma is to decrease f_0 until it reaches its minimum point then after that point diffraction term becomes dominant.

1. Introduction

The propagation of high-intensity light pulses in plasmas is of fundamental interest and essential to technological applications such as electron accelerators, x-ray lasers and fast-ignitor thermonuclear fusion [1]. Research into this problem has been possible within the last two decades, with the development of lasers capable of delivering pulses with field strengths sufficient to drive electrons to relativistic velocities. In most regimes of laser-plasma interactions, the laser beam suffering from optical diffraction which reduces the laser efficiency as a powerful tool in nonlinear phenomena such as laser fusion, laser acceleration and x-ray generation [2,3]. In this regime, the effects of relativistic quiver motion as well as laser pressure dominate the interaction, causing, among other things, relativistic self-focusing (RSF) and electron acceleration [2]. Since the amplitude of the light wave, in this regime, is very high, such that the light's magnetic field becomes comparable to that of the light's electric field which exceeds 10^{11} V/m, and the combined effect of these fields will make the plasma electrons oscillate at relativistic velocities, resulting in relativistic mass changes exceeding the electron rest mass, and hence, a relativistic nonlinear optics in the plasmas [4]. This nonlinear optics of plasmas is used in various applications, such as relativistic self-focusing [5], relativistic self-phase modulation, [6,7] Raman forward scattering, [8,9] relativistic harmonic generation, [10,11] laser driven accelerators, [12-14] x-ray lasers [15-17] and fast igniter fusion [18,19].

Relativistic self-focusing of the laser beams has been studied in detail both theoretically as well as experimentally [20-22]. The effects of pulse duration on the self-focusing of ultra-short pulse lasers in underdense plasma was investigated experimentally by Faure *et al.* [23] and they found that the laser pulse duration should be carefully compared to typical time scales of plasma particle motion. Hafizi *et al.* [24] have derived an envelope equation for the laser spot size to describe the axial evolution of the spot size as a function of the ratio of laser power P to the critical power P_c for relativistic focusing. Oscillation of the spot size is analytically described in terms of an effective potential. Sharma [25] has studied the steady state self focusing in plasma, immersed in a uniform magnetic field $\vec{B}_S$, perpendicular to the direction of electromagnetic wave propagation. The nonlinearity arises through the poderomotive force and subsequent redistribution of plasma along the magnetic field. As the magnetic field increases the extent of self

focusing increases. Borghesi et al. [26] have observed multimegagauss azimuthal magnetic-field generation with picosecond and sub-picosecond pulses of intensity ~10^{19} W/cm². Particle-in-cell simulations [27] have revealed the existence of strong dc magnetic fields (~100MG) that can strongly influence nonlinear phenomena. Liu and Tripathi [28] have shown that the azimuthal magnetic field has a focusing effect on rays in the plane of laser polarization. In this work, we study

2. Relativistic Self-Focusing of Intense Laser Pulse

The wave equation governing the electric field of the pump laser beam in plasma can be written as

$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial \vec{J}}{\partial t} \tag{1}$$

where $\vec{J}$ is the high-frequency current density, c is the velocity of light, and the term $\nabla(\nabla.\vec{E})$ has been neglected in Eq. (1). Substituting the value of $\vec{J}$ in the above equation and assuming the variation of the electric field as

$$\vec{E}(x,y,z) = A(x,y,z)\ e^{-i(\omega_0 t - kz)} \tag{2}$$

where A is the amplitude of the electric field, ω_0 is the angular frequency and k is the wave number. One obtains the following equation:

$$-k_0^2 A - 2ik_0 A + \nabla_\perp^2 A = -\frac{\omega_0^2}{c^2}\varepsilon A,$$

where $\quad \nabla_\perp^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$

According to this theory, the intensity-dependent permittivity of the plasma is

$$\varepsilon = 1 - \frac{\omega_p^2/\omega_0^2}{(1+\alpha EE^*)} \tag{3}$$

Moreover, the factor A, in Eq.(2), is a complex function of (x, y, z), further assuming the variation of A as given by Akhmanov et al. [29].

$$A = A_0(x,y,z)\ e^{-ik_0 S_0} \tag{4}$$

where A_0 and S_0 are the real functions of space (xyz), using the standard technique proposed by [29] for solving this kind of equations, i.e., substituting Eq. (4) into Eq. (2) and separating the real and imaginary parts of the resulting equation, the following set of equations are obtained:

The real part from Eq. (2) is

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{\varepsilon\omega_0^2}{c^2 k_0^2} + \frac{1}{k_0^2 A_0}\nabla_\perp^2 A_0 \tag{5}$$

where

$$A_0 = \frac{A_{00}^2}{f_0^2}e^{-r^2/2r_0^2 f_0^2} \tag{6}$$

is the laser beam intensity, f_0 is the beam width parameter, and

$$S = S_0/r_0^2 \tag{7}$$

with

$$S_0 = \frac{r_0^2}{f_0}\frac{df_0}{dz}$$

By substituting Eqs. (6), (7) and (3) into Eq. (5) and equating the coefficients of r^2 on both sides of the resulting equation, the equation governing the beam width parameter f_0 is obtained

$$\frac{d^2 f_0}{dz^2} = \frac{1}{k^2 r_0^4 f_0^3} - \frac{(\omega_p^2/\omega_0^2)\alpha_2 A_{00}^2}{2r_0^2 f_0^2} \tag{8}$$

where the first term in RHS is the diffraction term while the second term is the self-focusing term.

3. Effect of Transverse Magnetic Field on (RSF)

Consider a uniform plasma of equilibrium electron density n_0^0, immersed in a magnetic field $\vec{B}_S \| \hat{y}$. An intense laser beam propagates through it along $\hat{z}$ direction,

$$\vec{E}(x,y,z) = \vec{A}(x,y,z)\ e^{-i(\omega t - kz)} \tag{9}$$

The response of plasma electrons is governed by the relativistic equation of motion,

$$m\frac{\partial}{\partial t}(\gamma\vec{v}) + m\cdot\vec{\nabla}(\gamma\vec{v}) = -e\vec{E} - \frac{e}{c}\vec{v}\times\vec{B} \tag{10}$$

where $-e$ and m are the electronic charge and rest mass, c is the speed of light in vacuum and γ is the Lorentz factor. The x and z components of the velocity is given by

$$v_x = \left[\frac{-ieE_x}{m\omega_0\gamma} + \frac{\omega_c}{\omega_0\gamma}\frac{eE_z}{m\omega_0}\right]\left(1 - \frac{\omega_c^2}{\omega_0^2\gamma^2}\right)^{-1} \tag{11a}$$

$$v_z = \left[\frac{-ieE_x}{m\omega_0\gamma} + \frac{\omega_c}{\omega_0\gamma}\frac{eE_x}{m\omega_0\gamma}\right]\left(1 - \frac{\omega_c^2}{\omega_0^2\gamma^2}\right)^{-1} \tag{11b}$$

where $\vec{\omega}_c = e\vec{B}/mc$, and $\gamma \cong 1 + |v|^2/4c^2$

Using Eq. (11), one may obtain the current density $\vec{J} = \underline{\sigma}\vec{E} = n_0^0 e\vec{v}$ with $\underline{\sigma}$ as the conductivity tensor. The conductivity tensor $\underline{\sigma}$ have the following components

$$\sigma_{xx} = \sigma_{zz} = \frac{ine^2}{m\omega_0\gamma}\left(1 - \frac{\omega_c^2}{\omega_0^2\gamma^2}\right)^{-1} \tag{12}$$

$$\sigma_{xz} = -\sigma_{zx} = -\frac{ine^2}{m\omega_0\gamma}\left(\frac{\omega_c}{\omega_0\gamma}\right)\left(1 - \frac{\omega_c^2}{\omega_0^2\gamma^2}\right)^{-1} \tag{13}$$

The dielectric constant $\underline{\varepsilon}$ is also a tensor, its components are given by

$$\varepsilon_{zx} = -\varepsilon_{xz} = i\frac{\omega_{p0}^2\omega_{c0}/\gamma^2}{\omega(\omega^2 - \omega_{c0}^2/\gamma^2)} \tag{14}$$

$$\varepsilon_{xy} = \varepsilon_{yx} = \varepsilon_{zy} = \varepsilon_{yz} = 0$$

where $\omega_{p0} = \sqrt{4\pi n_0^0 e^2/m}$ and n_0^0 are the plasma frequency and the plasma density, respectively.

One may write

$$\gamma^2 = (1 - \frac{v^2}{c^2})^{-1} \cong 1 + \frac{v^2}{c^2} = 1 + \frac{v_x \cdot v_x^* + v_z \cdot v_z^*}{2c^2} \quad (15)$$

Substituting the values of v_x and v_z from Eq.(11), then the Lorentz factor can be written as

$$\gamma \cong 1 + \frac{e^2 E_x E_x^*}{4mc^2(\omega_0^2 - \omega_c^2)^2}\left[(\omega_0^2 + \omega_c^2) + (\omega_0^2 + \omega_c^2)\left|\frac{\varepsilon_{0xz}}{\varepsilon_{0xx}}\right|^2\right]$$

$$= 1 + \alpha|E_x|$$

where $\omega_{UH}^2 = \omega_p^2 + \omega_c^2$

let $\varepsilon_t = \varepsilon_0 + \varepsilon_2 E_x E_x^*$ be the total permittivity of the medium due to the linear and nonlinear terms, then it also can be given by

$$\varepsilon_t = 1 - \frac{\omega_p^2/\gamma}{\omega^2}\frac{\omega^2 - \omega_p^2/\gamma}{\omega^2 - \omega_p^2/\gamma - \omega_c^2/\gamma^2} \quad (17)$$

Using

$$\omega_p^2/\gamma = \omega_p^2/(1 + \alpha E_x E_x^*) \cong \omega_p^2(1 - \alpha E_x E_x^*) \quad (18a)$$

$$\omega_c^2/\gamma^2 = \omega_c^2/(1 + \alpha E_x E_x^*)^2 \cong \omega_c^2(1 - 2\alpha E_x E_x^*) \quad (18b)$$

and substituting Eq.(18) in Eq.(17), we obtain

$$\varepsilon_2 = \alpha E_x E_x^* \frac{\omega_p^2}{\omega_0^2}\frac{\omega_0^2 - \omega_p^2}{\omega_0^2 - \omega_p^2 - \omega_c^2}\left(\frac{(\omega_0^2 - \omega_p^2)^2 + \omega_0^2\omega_c^2}{(\omega_0^2 - \omega_{p0}^2)(\omega_0^2 - \omega_p^2 - \omega_c^2)}\right) \quad (19)$$

with $\omega_c = eB/mc = 0$ (there is no applied magnetic field), we get

$$\varepsilon_2 = \alpha E_x E_x^* \frac{\omega_p^2}{\omega_0^2} \quad (20)$$

4. Results and Discussion

At laser intensity $\geq 10^{17}$W/cm^2 and $\lambda_{\text{laser}} \approx 1\mu$m wavelength, the relativistic mass nonlinearity can cause self focusing of the laser beam when the plasma density $n \geq 0.01n_{cr}$, where n_{cr} is the critical density. The presence of magnetic field of a few tens of MG has significant favorable influence on the self focusing. In Fig. (1), we have calculated ε_2 as a function of normalized plasma frequency for four values of normalized cyclotronic frequency. The applied magnetic field has a profound desired effect on the nonlinear part of permittivity. Using an external magnetic field on laser-plasma interaction, one can increase the proportion of self-focusing effect.
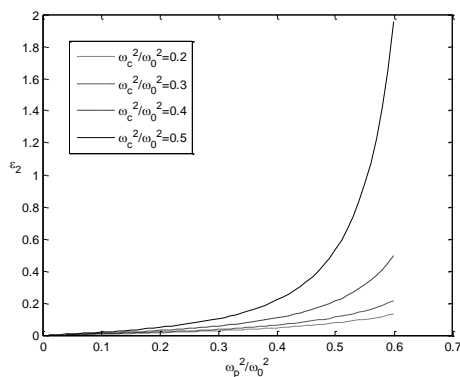
parameters: $k^2r_0^2 = 400$, and $\alpha|A_{00}|^2 = 0.3$, which is a function of laser intensity. Figure (2) shows that the beam width parameter initially decreases for all our values of normalized plasma plasma frequency, attains a minimum where the effect of the diffraction term becomes comparable to that of self-focusing term. Beyond the minimum point, the beam diverges because the diffraction term in Eq. (8) becomes more predominant than the self focusing term., the minimum point is lowest for the value of $\omega_p^2/\omega_0^2 = 0.06$.

Figure (3) is plot the of beam width parameter f_0 as a function of normalized cyclotronic frequency for four values of normalized plasma frequency for $\alpha|A_{00}|^2 = 0.2$, $k^2r_0^2 = 400$. For all the plotted values of ω_{po}^2/ω^2 (which a function of plasma density) the beam width parameter decreases as ω_{po}^2/ω^2 increases. The extent of self-focusing depend also on the wavelength of laser. In Fig. (4), we have plotted f_0 as a function of ω_c/ω_0 for some selected lasers' wavelengths. The figure demonstrates that the most effective laser is the one with shortest wavelength (i.e., $\lambda = 0.488\mu m$).
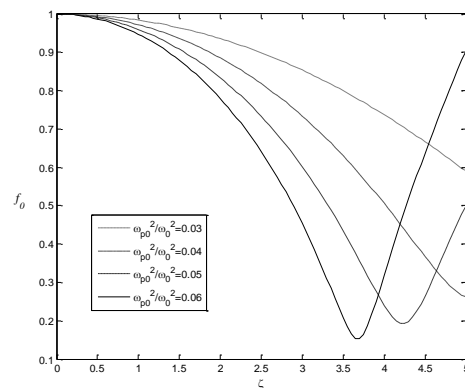

Fig. (1) The nonlinear effective permittivity ε_2 as a function of ω^2/ω_{po}^2 for $\alpha|A_{00}|^2=0.4$, $k^2r_0^2=400$

To get a numerical result of this effect on the laser beam width parameter, we have solved Eq. (8) using Runge-Kutta method. Figure (2) shows the variation of laser beam width parameter f_0 with the distance of propagation $\zeta = z/kr_0^2$ for the following

Fig. (2) Variation of beam width parameter f_0 as a function of distance ζ for four values of cyclotronic frequency
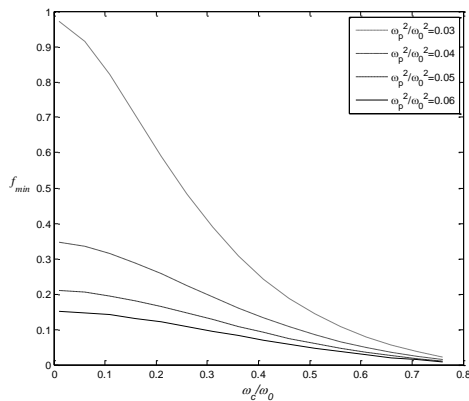
Fig. (3) Variation of minimum beam width parameter f_{min} as a function of normalized cyclotronic frequency for four values of normalized plasma frequency
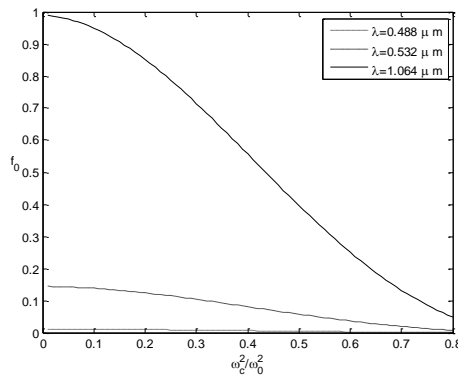

Fig. (4) Variation of beam width parameter f_0 as a function of normalized cyclotronic frequency for four values of laser wavelength

At 1µm wavelength, $\omega_c \approx \omega_0$ occurs at B_s=100MG and such a magnetic fields have been observed experimentally [26]. When magnetic field exceeds the 10MG, the extent of self focusing increases. Also the beam converges more rapidly and focuses up to a smaller spot size. The term f_0 decreases monotonically with ω_c/ω_0. Also, it is also decreases with increasing plasma density.

5. Conclusion

Applying an external transverse magnetic field on a laser beam interacting with plasma has a desired effect in reducing the minimum beam width parameter. However this effect is limited to a certain propagation length, where the diffraction becomes more important than the self-focusing term.

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